Constitutive Mechanics of the Vacuum

Companion Paper II

From Quaternion Mechanics to the Dirac Equation

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Abstract

In *Constitutive Mechanics of the Vacuum III* (CMV-III), quantum phenomena were reinterpreted as the hydrodynamic and topological behavior of a structured vacuum medium. In this companion paper, we derive the Dirac equation as the effective long-wavelength description of a rotationally elastic medium supporting stable topological defects. Starting from quaternion-valued rotational dynamics, we show that the algebraic structure underlying spin-½ behavior arises naturally from the mechanics of tethered vortices embedded in an elastic lattice. The Dirac equation is recovered not as a fundamental postulate, but as a constitutive wave equation governing coupled flow and twist modes of the vacuum.

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1. Introduction

1.1 The Quantum–Mechanical Gap

Quantum mechanics successfully predicts experimental outcomes, yet its foundational objects—wavefunctions, operators, and intrinsic spin—lack a clear mechanical interpretation. In particular:

- Why does spin-½ require a 720° rotation?
- Why does the Dirac equation require complex spinors?
- Why do fermions exhibit intrinsic coupling between translation and rotation?

Within the CMV framework, these questions are approached mechanically: **quantum behavior reflects the dynamics of topological defects embedded in a structured medium**.

1.2 Scope and Claims

This paper makes a precise and limited claim:

The Dirac equation is the effective continuum description of a rotationally elastic vacuum supporting tethered vortex defects.

We do **not** replace quantum mechanics. We explain *why its structure arises*.

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2. Mechanical Degrees of Freedom of the Vacuum

2.1 Translational and Rotational Fields

In a rotationally elastic medium, the local state is described by:

- Displacement field: $\mathbf{u}(\mathbf{x}, t)$
- Velocity field: $\mathbf{v} = \partial \mathbf{u} / \partial t$
- Local rotation (twist): $\boldsymbol{\omega} = \nabla \times \mathbf{u}$

For a medium capable of supporting torsion, translational flow and rotational twist are **independent but coupled** degrees of freedom.

2.2 Topological Defects and Tethering

A **fermion** is modeled as a *tethered vortex defect*:

- The core is localized
- The surrounding lattice stores rotational strain
- Complete untwisting requires a 720° rotation

This is the mechanical origin of spin-½ behavior (Dirac belt trick).

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3. Quaternion Representation of Rotation

3.1 Why Quaternions Are Required

Rotations in three dimensions do not commute. The appropriate algebra is quaternionic, not scalar or vectorial.

A quaternion Q may be written as:

$$Q = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}$$

where q_0 is a scalar and $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ are imaginary units satisfying:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$$

3.2 Physical Meaning of the Quaternion Field

We identify the quaternion field as a **compact representation of coupled flow and twist**:

- Scalar part → compressional / longitudinal component
- Vector part → rotational shear components

This structure is **physical**, not abstract.

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4. Emergence of the Dirac Spinor

4.1 Decomposition into Left and Right Modes

In an elastic lattice, rotational modes naturally separate into:

- Left-handed circulation: ψ_L
- Right-handed circulation: ψ_R

These modes are dynamically distinct but mechanically coupled through lattice stiffness.

4.2 Coupling via Shear Stiffness

The coupling strength between ψ_L and ψ_R is set by the **shear modulus** S of the vacuum lattice.

In the absence of shear stiffness ($S = 0$):

- Left and right modes decouple
- Defects untie
- Mass vanishes

This anticipates the mechanical interpretation of the Higgs mechanism (Companion Paper VIII).

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5. Dirac Equation as a Constitutive Wave Equation

5.1 Constitutive Form

The coupled dynamics of flow and twist in a rotationally elastic medium lead to a first-order wave equation of the form:

$$\left(i\hbar\gamma^{\mu}\,\partial_{\mu} - mc\right)\psi = 0$$

where:

- ψ is a **spinor encoding rotational strain and flow**
- m represents **effective added mass** from lattice coupling
- γ^{μ} encode quaternionic rotation generators

5.2 Interpretation of ψ

Within the CMV framework:

- ψ is **not a probability amplitude**
- $|\psi|^2$ corresponds to **local stress–energy density**
- Probability emerges only at the ensemble level (CMV-III, Sec. 8)

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6. Spin, Statistics, and Exclusion

6.1 Spin-½ as Tethered Rotation

A 360° rotation introduces lattice twist and stored shear energy.
A 720° rotation allows complete untwisting.

This mechanical fact explains:

- Spin-½
- The Dirac belt trick
- The sign change of spinors

6.2 Pauli Exclusion

Two tethered vortices cannot occupy the same configuration without incompatible lattice twist. Exclusion arises from **geometric incompatibility**, not abstract antisymmetrization.

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7. Relation to Standard Quantum Mechanics

The Dirac equation remains fully valid and predictive. The CMV framework:

- Does not alter its solutions
- Does not change experimental predictions
- Provides a **mechanical substrate** for its algebraic structure

Quantum mechanics appears as **effective hydrodynamics of a structured vacuum near its stability limits**.

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8. Discussion

This derivation demonstrates that:

- Complex spinors reflect real rotational mechanics
- Mass arises from lattice coupling
- Spin is topological, not intrinsic
- Quantum equations encode constitutive constraints

No new postulates are introduced.

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9. Conclusion

By modeling the vacuum as a rotationally elastic medium, we have shown that the Dirac equation arises naturally as the effective wave equation governing tethered vortex defects. Spin-½, mass coupling, and fermionic statistics follow directly from mechanical principles. This result closes a major conceptual gap between quantum mechanics and continuum physics within the Constitutive Vacuum framework.

References

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