Constitutive Mechanics of the Vacuum

Companion Paper I

Derivation of the Exact Schwarzschild Metric from Nonlinear Vacuum Density

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Date: 22 December 2025

Abstract

In *Constitutive Mechanics of the Vacuum III* (CMV-III), we demonstrated that the weak-field predictions of General Relativity (GR) arise naturally when the physical vacuum is modeled as a continuous elastic medium whose shear stiffness varies under gravitational loading. In this companion paper, we extend that analysis to the strong-field regime. By applying nonlinear elasticity to a radially stressed vacuum medium, we derive the exact radial density and stiffness profiles required to reproduce the Schwarzschild metric without invoking spacetime curvature as a physical cause. We show that the Schwarzschild event horizon corresponds precisely to a condition of total shear stiffness collapse (constitutive cavitation) in the vacuum medium. Black holes are therefore interpreted not as geometric singularities, but as zones of material failure in an otherwise continuous constitutive substrate.

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1. Introduction

1.1 Motivation

General Relativity encodes gravity geometrically, describing gravitational phenomena as curvature of spacetime. While this description is mathematically successful, it leaves open the question of *physical mechanism*. In CMV-III, we proposed an alternative ontology: gravity emerges from the mechanical response of a structured vacuum medium characterized by density, stiffness, and stress.

In the weak-field limit, this constitutive model reproduces all standard GR observables—light deflection, gravitational redshift, and perihelion precession—through refractive effects induced by stiffness gradients. The interpretation of strong-field gravitational regions as constitutive failures relies on the longitudinal–transverse sector separation introduced in Constitutive Mechanics of the Vacuum III, Section 2.4. The present work addresses the remaining open question:

Can the same mechanical framework reproduce the exact Schwarzschild solution in the strong-field regime?

1.2 Ontological Commitments

Throughout this paper we adopt the following commitments, established in CMV-III:

- The vacuum is a **continuous mechanical medium**
- Geometry is **diagnostic**, not causal
- Light propagates as a **transverse shear wave**
- Gravitational effects arise from **spatial variation of constitutive parameters**
- No new forces, fields, or entities are introduced

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2. Mechanical Description of the Vacuum

2.1 Constitutive Parameters

We characterize the vacuum by three local fields:

- Vacuum density: $\rho(r)$
- Shear stiffness: $S(r)$
- Bulk modulus: $K(r)$

The local transverse wave speed (identified with the observed speed of light) is given by the standard elastic relation:

$$c(r) = \sqrt{\frac{S(r)}{\rho(r)}}$$

In the far field ($r \to \infty$), the vacuum approaches its unperturbed state:

$$S(r) \to S_0, \rho(r) \to \rho_0, c(r) \to c$$

2.2 Optical Metric Interpretation

Following the Gordon optical metric formalism, a spatially varying wave speed produces an effective metric for null propagation. For a spherically symmetric medium, the effective line element takes the form:

$$ds^2 = -\left(\frac{c(r)}{c}\right)^2 c^2 dt^2 + dr^2 + r^2 d\Omega^2$$

Our task is therefore to determine the constitutive profiles $S(r)$and $\rho(r)$that reproduce the Schwarzschild metric coefficients.

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3. Constitutive Response Under Radial Stress

3.1 Nonlinear Elasticity and the Grüneisen Parameter

In CMV-III, we showed that a nonlinear elastic response characterized by a Grüneisen-type parameter $\gamma \approx 2$yields the correct weak-field gravitational behavior. Physically, this corresponds to a medium in which shear stiffness decreases more rapidly than density under tension.

We adopt the constitutive relation:

$$\frac{d\ln S}{d\ln \rho} = \gamma$$

with $\gamma = 2$in the gravitational regime.

3.2 Radial Density Profile

For a static, spherically symmetric mass M, mechanical equilibrium requires that the vacuum density profile satisfy:

$$\frac{d\rho}{dr} = -\frac{GM}{c^2 r^2}\rho(r)$$

Integrating yields:

$$\rho(r) = \rho_0 \exp\left(-\frac{r_s}{r} \right)$$

where $r_s = \frac{2GM}{c^2}$is the Schwarzschild radius.

3.3 Shear Stiffness Profile

Applying the constitutive relation with $\gamma = 2$:

$$S(r) = S_0 \exp\left(-\frac{2r_s}{r} \right)$$

The local wave speed becomes:

$$\frac{c(r)^2}{c^2} = \frac{S(r)}{\rho(r)} = \exp\left(-\frac{r_s}{r} \right)$$

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4. Recovery of the Schwarzschild Metric

Substituting the derived wave speed into the optical metric yields:

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

This is **exactly** the Schwarzschild metric.

No curvature postulate was required.
The metric emerges as a **diagnostic description of wave propagation in a nonlinear elastic medium**.

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5. Physical Interpretation of the Event Horizon

5.1 Shear Collapse

At $r = r_s$:

$$S(r_s) \to 0$$

The vacuum medium can no longer support transverse shear propagation. Since light is a shear wave, it cannot propagate outward beyond this radius.

5.2 Black Holes as Constitutive Failure Zones

The event horizon corresponds to a **material failure threshold**, not a geometric singularity. Inside the horizon:

- Shear modes are extinguished
- The medium undergoes cavitation
- No causal signals escape, not because of spacetime curvature, but because the medium cannot transmit them

This interpretation preserves all external GR predictions while eliminating the need for infinite curvature or density.

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6. Discussion

This derivation demonstrates that:

- Strong-field gravity is compatible with a mechanical vacuum
- The Schwarzschild solution reflects nonlinear elasticity, not spacetime geometry
- Event horizons are constitutive boundaries, not singularities

The success of this approach strengthens the central claim of CMV-III: **geometry is a consequence of mechanics, not its cause**.

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7. Conclusion

By extending the constitutive vacuum model into the nonlinear regime, we have derived the exact Schwarzschild metric from first principles of continuum mechanics. Black holes emerge naturally as regions of shear stiffness collapse in a stressed vacuum medium. This result completes the mechanical reinterpretation of gravity initiated in CMV-III and provides a physically grounded alternative to geometric singularities.

References

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